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## Neutrino Decoupling and its Consequences:

So far we have considered what happens to baryons after the weak interactions freeze out at $t \sim 1\,sec$ ($T \sim 1\,MeV$).

To give a brief summary:

1- $t \sim 1\,sec$: weak interactions freeze out. At this time $\frac{n}{p}$ is given by the equilibrium value $\exp(-\frac{m_n - m_p}{T})$. The only interaction that proceeds efficiently is neutron decay $n \to p + e^- + \bar{\nu}_e$ henceforth.

2- $1\,sec < t \sim 100\,sec$: the Deuterium bottleneck. The protons and neutrons combine to form $^2D$, but the energetic photons in the Wien tale of CMB black body spectrum are copious and photodissociate $^2D$.

3- $100\,sec \lesssim t \lesssim 300\,sec$: synthesis of light elements. $^2D$ can be formed, which subsequently participates in nuclear reactions to form $^3He$, $^4He$, $^7Li$. Almost all of the neutrons end up in $^4He$, and a tiny number of $^2D$, $^3He$ (even a smaller number of $^7Li$) remain to be around.

Now let us to turn to leptons and see what happens to them after 1 sec. The only leptons (in the relativistic regime) then are $e^{\pm}$ and the three types of neutrinos $\nu_e, \nu_\mu, \nu_\tau$ (and their antiparticles).

At $T \sim 1\,\mathrm{MeV}$ we have:

$$g_* = \underbrace{2}_{\gamma} + \underbrace{\frac{7}{8} \times 4}_{e^{\pm}} + \underbrace{\frac{7}{8} \times 2 \times 3}_{\substack{\nu_e, \nu_\mu, \nu_\tau \\ \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau}}$$

At this temperature the plasma still consists of photons, $e^{\pm}$, and neutrinos/antineutrinos all in thermal equilibrium.

However, at $t > 1\,\mathrm{sec}$ ($T < 1\,\mathrm{MeV}$) there will be two components that cannot communicate with each other anymore (note that neutrinos have only weak interactions): $\gamma$ and $e^{\pm}$, neutrinos/antineutrinos

Once $T$ drops below $0.5\,\mathrm{MeV}$ (mass of the electron) then $e^{\pm}$ pairs annihilate to photons. For $T \ll 0.5\,\mathrm{MeV}$, the inverse process is inefficient because of the small number of energetic photons. The $e^{+} + e^{-} \to \gamma + \gamma$ annihilation process is very efficient because it happens via electromagnetic interactions. As a result essentially all $e^{\pm}$ in the plasma disappear. Note that, due to efficiency of $e^{\pm}$ annihilation, this happens adiabatically. In consequence, the total entropy remains constant.

Now let us consider the times "before" and "after" the annihilation of $e^{\pm}$. Before that happens we have:

$$s_{before} = \frac{2\pi^2}{45} g_*^{before} T_{before}^3$$

And after the annihilation completes:

$$s_{after} = \frac{2\pi^2}{45} g_*^{after} T_{after}^3$$

Here we consider only $e^{\pm}, \gamma$ since neutrinos have decoupled at $t \sim 1$ sec. Adiabaticity of the annihilation process requires that:

$$s_{before} = s_{after}$$

Here we use the comoving quantities of the entropy density "s" and temperature "T". Note that conservation of entropy $S = sV$ implies the conservation of comoving entropy density (whic is just the entropy itself).

On the other hand:

$g_*^{before} = 2 + \frac{7}{8} \times 4 = \frac{11}{2}$ $(\gamma, e^{\pm})$

$g_*^{after} = 2$ $(\gamma$ only$)$

Thus:

$$g_*^{before} \left(T^c_{\gamma, before}\right)^3 = g_*^{after} \left(T^c_{\gamma, after}\right)^3$$

Where the superscript "c" denotes comoving value.

We then find:

$$T^c_{\gamma, before} = T^c_{\gamma, after} \left(\frac{11}{4}\right)^{-1/3} \Rightarrow \boxed{T^c_{\gamma, after} = \left(\frac{4}{11}\right)^{-\frac{1}{3}} T^c_{\gamma, before}}$$

Note that if $g_*$ was the same, then $T_\gamma^c$ would remain unchanged. However, a change in $g_*$ result in a change in $T_\gamma^c$. Since $g_*$ decreases, photons get heated up. On the other hand, neutrinos will not know about this since the weak interactions have already frozen. This implies that:

$$\boxed{T_\nu^c = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma^c \qquad (T \ll 1\,\text{MeV})}$$

To put it in perspective, after the annihilation of $e^\pm$ has completed, we have:

$$n_\gamma^c = \frac{\zeta(3)}{\pi^2} \times 2 \times (T_\gamma^c)^3$$

$$n_\nu^c = \frac{\zeta(3)}{\pi^2} \times \frac{3}{4} \times 2 \times (T_\nu^c)^3 \qquad \text{(neutrinos and antineutrinos per family)}$$

Hence:

$$\boxed{\frac{n_\nu^c}{n_\gamma^c} = \frac{3}{4} \times \frac{4}{11} \Rightarrow n_\nu^c = \frac{3}{11} n_\gamma^c}$$

The physical number density of photons at the present time is given by:

$$\boxed{n_{\gamma,0} = \frac{\zeta(3)}{\pi^2} \times 2 \times T_0^3 \sim 420\ \text{cm}^{-3}} \qquad (T_0 \sim 0.0003\ \text{eV})$$

While, for each neutrino family:

$$n_{\nu,0} = \frac{\zeta(3)}{\pi^2} \times \frac{3}{4} \times 2\, T_0^3 \times \frac{4}{11} \sim 114\ \text{cm}^{-3}$$

Even though we considered the change in the comoving temperature of photons after the annihilation of $e^{\mp}$, the same thing happens every time the plasma temperature drops below the mass of a massive particle.

At temperature above its mass, that particle is in the relativistic regime and in thermal equilibrium with the rest of the plasma. Once the temperature drops below its mass, the particle becomes non-relativistic. If it is unstable, it will mainly decay afterward to lighter degrees of freedom. The inverse decay will not be efficient due to kineamatic reasons. If the particle is stable it will annihilate with its antiparticle to lighter degrees of freedom (again the inverse process is not efficient because of kinematics). So long as decay/annihilation is efficient (i.e., its rate is greater than the Hubble expansion rate) the process happens adiabatically, and hence the entropy (or the comoving entropy density) will remain constant. Because of the change in the number of relativistic degrees of freedom (a decrease) this conservation requires that the comoving temperature of the plasma change too (an increase).

To elucidate, lets consider to temperatures $T_1 \sim 200$ GeV and $T_2 \sim 1$ MeV. At $T_1$ all of the standard model degrees of freedom are relativistic and in thermal equilibrium. Then,

$s_1^c = g_{*,1} (T_1^c)^3$ $\qquad g_{*,1} = 106.75$

On the other hand, as we just saw:

$s_2^c = g_{*,2} \left(T_2^c\right)^3 \qquad g_{*,2} = 10.75$

Thus:

$$\boxed{T_2^c = T_1^c \left(\frac{106.75}{10.75}\right)^{1/3} \Rightarrow T_2^c = 2.15\, T_1^c}$$

Note that $T_2^c = T_1^c$ had there been no change in $g_*$.

Now assume there exists a new degree of freedom (for example a particle beyond the standard model) that has very weak interactions with the standard model particles and decouples from the plasma at $T \sim 200$ GeV.

Then, it will not notice the increase in the comoving temperature of the plasma because of the decrease in $g_*$. As a result, its comoving temperature will remain constant at $T_1^c$. At a physical temperature $T \sim 1$ MeV, the total energy density of the plasma is:

$$\rho = \frac{\pi^2}{30} g_* T^4 + \frac{\pi^2}{30} T^4 \times (2.15)^{-4}$$

Here we have assumed the new particle beyond the standard model represents one degree of freedom. The factor $(2.15)^{-4}$ arises because of the difference between the temperature of that particle and the plasma temperature. Recall that $g_* = 10.75$ at $T \sim 1$ MeV.

We therefore find:

$$\boxed{\rho = \frac{\pi^2}{30}\left[g_* + \left(\frac{1}{2.15}\right)^4\right] T^4 = \frac{\pi^2}{30} \times 10.80 \times T^4}$$

The new hypothetical particle shows up as 0.05 effective degree of freedom. It is important to note that it represents one physical degree of freedom, but the fractional number comes because of its much lower temperature

Finally, we note that the comoving entropy density is constant most of the time in the early universe. (The main exception is after inflation when there is no entropy, and reheating creates all particles and a huge entropy.) The physical entropy density is therefore redshifted $\propto V^{-1}$ ($V$ being the volume). Hence any number density normalized by the entropy density does not feel any change due to the Hubble expansion.

However, the physical number density of photons is only good for the normalization purpose after the $e^{\pm}$ annihilation completes (i.e. at $T \ll 1$ MeV). The reason being that the comoving number density of photons increases each time a massive particle decays or annihilates.